Yeom, Samuel et al. "Overfitting, robustness, and malicious algorithms: A study of potential causes of privacy risk in machine learning"

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### Purpose

- Machine Learning(ML) is used to solve a wide range of problems
- This includes problems where data may be sensitive, i.e. healthcare
- Ideally, we would not want a member of the data set to be identified, or more information about them to be known
  - Training Set Member Inference
  - Attribute Inference
- The paper analyzed factors of ML algorithms such as overfitting, robustness, and malicious algorithms and their negative effect on privacy of machine learning algorithms

# Background/Preliminaries

## **Background Terms**

- **Overfitting**: A ML model is said to overfit when it fits too closely with a certain dataset
- **Training Set Member Inference:** Determine whether a given data point was present in the training set
- Attribute Inference: An adversary uses a ML model and incomplete information about a data point to infer the missing information for that point
- **Robustness:** A measure of how resilient ML models are to adversarial perturbations to the input data



### **Preliminaries - Definitions and Notation**

- Data Point:  $z = (x, y) \in \mathbf{X} \times \mathbf{Y}$
- $z \sim S$ : *i* is picked uniformly at random from [*n*], and *z* is set equal to the *i*-th element of *S*.  $z \sim D$ : *z* is chosen according to the distribution D.
- A<sub>s</sub> means a model A trained on dataset S
- Loss Function:  $\ell(A_S, z)$



### **Preliminaries - Stability**

• **Stable:** An algorithm is said to be stable if a small change to its input causes limited changes to its output.



### **Preliminaries - Differential Privacy**



#### **Preliminaries - Average Generalization Error**

**Definition 3** (Average generalization error). The *average generalization error* of a machine learning algorithm A on  $\mathcal{D}$  is defined as

$$R_{\text{gen}}(A, n, \mathcal{D}, \ell) = \mathbb{E}_{\substack{S \sim \mathcal{D}^n \\ z \sim \mathcal{D}}} \left[ \ell(A_S, z) \right] - \mathbb{E}_{\substack{S \sim \mathcal{D}^n \\ z \sim S}} \left[ \ell(A_S, z) \right].$$



### **Observations**

- Stability and Differential Privacy are closely linked
- Unstable algorithms may lead to high average generalization error, which means overfitting
- Unstable, and overfit algorithms may violate differential privacy thresholds



## Membership Inference Attacks

### **Formal Definition**

**Experiment 1** (Membership experiment  $\text{Exp}^{M}(\mathcal{A}, A, n, \mathcal{D})$ ). Let  $\mathcal{A}$  be an adversary, A be a learning algorithm, n be a positive integer, and  $\mathcal{D}$  be a distribution over data points (x, y). The membership experiment proceeds as follows:

- (1) Sample  $S \sim \mathcal{D}^n$ , and let  $A_S = A(S)$ .
- (2) Choose  $b \leftarrow \{0, 1\}$  uniformly at random.
- (3) Draw  $z \sim S$  if b = 0, or  $z \sim D$  if b = 1
- (4)  $\operatorname{Exp}^{M}(\mathcal{A}, A, n, \mathcal{D})$  is 1 if  $\mathcal{A}(z, A_{S}, n, \mathcal{D}) = b$  and 0 otherwise.  $\mathcal{A}$  must output either 0 or 1.

**Definition 4** (Membership advantage). The membership advantage of A is defined as

$$\mathsf{Adv}^{\mathsf{M}} = \Pr[\mathcal{A} = 0 \mid b = 0] - \Pr[\mathcal{A} = 0 \mid b = 1],$$

### **Bounded Loss Function Adversary**

Adversary 1 (Bounded loss function). Suppose  $\ell(A_S, z) \leq B$  for some constant B, all  $S \sim D^n$ , and all z sampled from S or D. Then, on input z = (x, y),  $A_S$ , n, and D, the membership adversary A proceeds as follows:

- (1) Query the model to get  $A_S(x)$ .
- (2) Output 1 with probability  $\ell(A_S, z)/B$ . Else, output 0.

Theorem 2:

$$\operatorname{Adv}^{M}(\mathcal{A}, A, n, \mathcal{D}) = R_{\operatorname{gen}}(A)/B$$

### (Gaussian) Threshold Adversaries

Adversary 2 (Threshold). Suppose  $f(\epsilon | b = 0)$  and  $f(\epsilon | b = 1)$ , the conditional probability density functions of the error, are known in advance. Then, on input z = (x, y),  $A_S$ , n, and D, the membership adversary A proceeds as follows:

- (1) Query the model to get  $A_S(x)$ .
- (2) Let  $\epsilon = y A_{\mathcal{S}}(x)$ . Output  $\arg \max_{b \in \{0,1\}} f(\epsilon \mid b)$ .

Advantage given by the ratio of standard errors:

$$\sigma_{\mathcal{D}}/\sigma_{S}$$



### **Unknown Standard Error Adversaries**

- Common for only one value for standard error given
- Solution: Assume they are roughly the same (not overfitting)
- Or, if type of ML algorithm is known: approximate the standard error of S and D by repeatedly sampling S from D<sup>n</sup>, train Algorithm A<sub>S</sub> and measure the error



#### **Malicious Adversaries**

Algorithm 1 (Colluding training algorithm  $A^{c}$ ). Let  $F_{K} : \mathbf{X} \mapsto \mathbf{X}$  and  $G_{K} : \mathbf{X} \mapsto \mathbf{Y}$  be keyed pseudorandom functions,  $K_{1}, \ldots, K_{k}$  be uniformly chosen keys, and A be a training algorithm. On receiving a training set S,  $A^{c}$  proceeds as follows:

- (1) Supplement S using F, G: for all (x<sub>i</sub>, y<sub>i</sub>) ∈ S and j ∈ [k], let z'<sub>i,j</sub> = (F<sub>Kj</sub>(x<sub>i</sub>), G<sub>Kj</sub>(x<sub>i</sub>)), and set S' = S ∪ {z'<sub>i,j</sub> | i ∈ [n], j ∈ [k]}.
  (2) Peture A = A(S')
- (2) Return  $A_{S'} = A(S')$ .

Adversary 3 (Colluding adversary  $\mathcal{A}^{C}$ ). Let  $F_{K} : \mathbf{X} \mapsto \mathbf{X}$ ,  $G_{K} : \mathbf{X} \mapsto \mathbf{Y}$  and  $K_{1}, \ldots, K_{k}$  be the functions and keys used by  $A^{C}$ , and  $A_{S'}$  be the product of training with  $A^{C}$  with those keys. On input z = (x, y), the adversary  $\mathcal{A}^{C}$  proceeds as follows:

- (1) For  $j \in [k]$ , let  $y'_j \leftarrow A_{S'}(F_{K_j}(x))$ .
- (2) Output 0 if  $y'_j = G_{K_j}(x)$  for all  $j \in [k]$ . Else, output 1.

## Attribute Inference Attack

### Notation Update !

- z is now a triple z = (v, t, y) where (v, t)  $\in$  X, and t is a sensitive feature
- $\varphi(z)$  is a function that describes the data known to the adversary (v, t)
- T is the support of t
- $\pi(z) = t$  is the projection of X into T



### **Formal Definition**

**Experiment 2** (Attribute experiment  $\text{Exp}^{A}(\mathcal{A}, A, n, \mathcal{D})$ ). Let  $\mathcal{A}$  be an adversary, *n* be a positive integer, and  $\mathcal{D}$  be a distribution over data points (x, y). The attribute experiment proceeds as follows:

- (1) Sample  $S \sim \mathcal{D}^n$ .
- (2) Choose  $b \leftarrow \{0, 1\}$  uniformly at random.
- (3) Draw  $z \sim S$  if b = 0, or  $z \sim D$  if b = 1.
- (4)  $\operatorname{Exp}^{A}(\mathcal{A}, A, n, \mathcal{D})$  is 1 if  $\mathcal{A}(\varphi(z), A_{S}, n, \mathcal{D}) = \pi(z)$  and 0 otherwise.

$$\mathsf{Adv}^{\mathsf{A}} = \sum_{t_i \in \mathbf{T}} \Pr_{z \sim \mathcal{D}}[t = t_i] \big( \Pr[\mathcal{A} = t_i \mid b = 0, t = t_i] - \Pr[\mathcal{A} = t_i \mid b = 1, t = t_i] \big),$$

#### **General Attribute Inference Adversary**

Adversary 4 (General). Let  $f_{\mathcal{A}}(\epsilon)$  be the adversary's guess for the probability density of the error  $\epsilon = y - A_S(x)$ . On input v, y,  $A_S$ , n, and  $\mathcal{D}$ , the adversary proceeds as follows:

- (1) Query the model to get  $A_S(v, t_i)$  for all  $i \in [m]$ .
- (2) Let  $\epsilon(t_i) = y A_S(v, t_i)$ .
- (3) Return the result of  $\arg \max_{t_i} (\Pr_{z \sim D}[t = t_i] \cdot f_{\mathcal{A}}(\epsilon(t_i))).$

$$\mathsf{Adv}^{\mathsf{A}} = \sum_{t_i \in \mathbf{T}} \Pr_{z \sim \mathcal{D}} [t = t_i] \big( \Pr[\mathcal{A} = t_i \mid b = 0, t = t_i] - \Pr[\mathcal{A} = t_i \mid b = 1, t = t_i] \big),$$



## Membership Inference on Robust Models

### **Robust Classification**

**Experiment 1** (Membership experiment  $\text{Exp}^{M}(\mathcal{A}, A, n, \mathcal{D})$ ). Let  $\mathcal{A}$  be an adversary, A be a learning algorithm, n be a positive integer, and  $\mathcal{D}$  be a distribution over data points (x, y). The membership experiment proceeds as follows:

- (1) Sample  $S \sim \mathcal{D}^n$ , and let  $A_S = A(S)$ .
- (2) Choose  $b \leftarrow \{0, 1\}$  uniformly at random.
- (3) Draw  $z \sim S$  if b = 0, or  $z \sim D$  if b = 1
- (4)  $\operatorname{Exp}^{\mathsf{M}}(\mathcal{A}, A, n, \mathcal{D})$  is 1 if  $\mathcal{A}(z, A_S, n, \mathcal{D}) = b$  and 0 otherwise.  $\mathcal{A}$  must output either 0 or 1.

$$Adv^{M} = Pr[\mathcal{A} = 0 | b = 0] - Pr[\mathcal{A} = 0 | b = 1],$$

Adversary 8 (Robust classification). Suppose  $A_S$  is a robust classification model with robustness parameter  $\rho$ . On input z = (x, y),  $A_S$ , n, and D, the membership adversary A proceeds as follows:

- (1) Find a perturbed input x' such that  $d(x, x') \leq \rho$ .
- (2) Query the model to get  $A_S(x')$ .
- (3) *Output*  $\ell(A_S, (x', y))$ .

Conclusion

## Summary of Findings Covered and Not Covered

- Introduced several new definitions of advantage both membership and attribute inference attacks
- Showed theoretically (and experimentally) that the more a model is overfit the more vulnerable it is to these types of attacks
- Stable, colluding training algorithms can be built for CNNs meaning that privacy can be leaked
- Robustness can be a source of membership advantage
- (Not Covered) They proved that there is a reduction between membership and attribute inference attacks and vice versa
- (Not Covered) Experimentally proven

## More Stuff Not Covered Here

## Would I accept this paper?

- I think that this is an interesting paper that shows with convincing formal proofs, and experimental results that these factors can affect algorithm privacy
- Making machines private was well understood, but the precise factors inside ML algorithms that could lead to privacy risks were not well studied



### Reductions

• Membership and Attribute inferences can be reduced to each other



### **Experimental Results**

• Confirm the theoretical results of the paper



Questions?

## **Discussion Questions**

- In the tradeoff of robustness vs. member privacy, what is more important? What are real world examples to support your claim?
- 2. Do you feel these observations are significant? Would you accept this paper?
- 3. The authors made a lot of assumptions about knowledge of the model/access. Do you feel like the scenarios studied are likely enough to happen? Or are they contrived?

