Lecture 5 | Kalman Filters

Secure Autonomous Systems

CSCI 6907/3907 88

Fall 2022

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https://bit.ly/secureauto-fall22

Kalman Filter

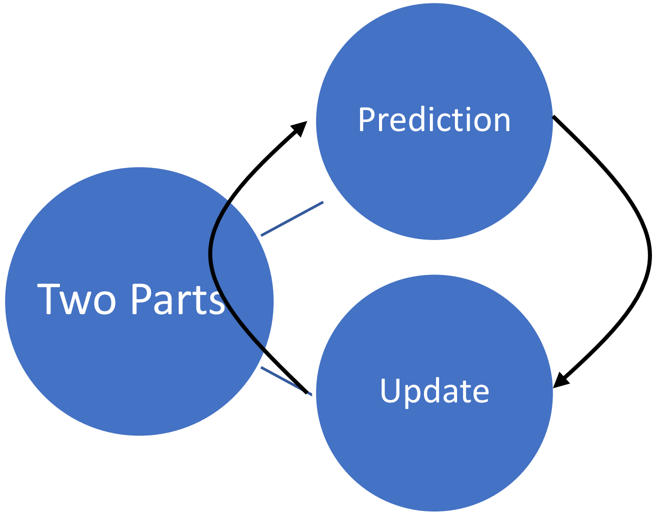
- For data fusion ➔ estimate the state of a dynamic system
  - In the present (filtering)
  - past (smoothing)
  - future (prediction)

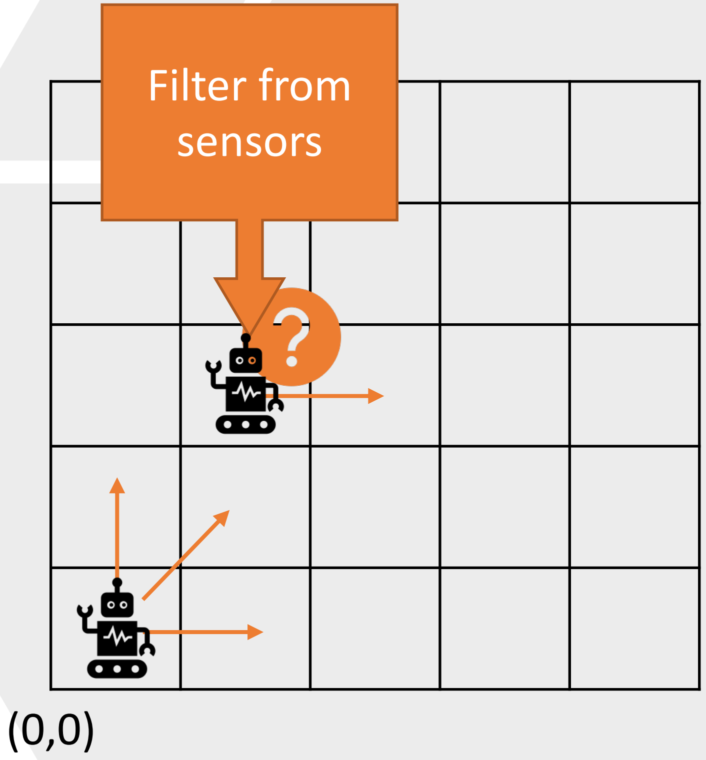
- Estimate the **state** of a robot from odometry data+observations

- Bayesian Filter

Bayes Filter

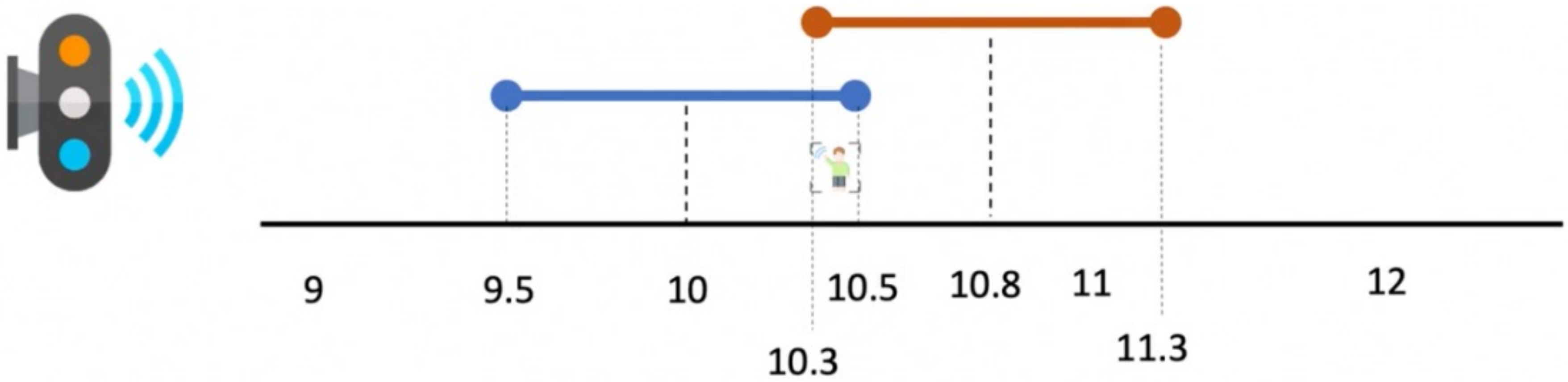
- probabilistic approach
- estimating an **unknown PDF**
  - recursively over time using
  - incoming **measurements** and
  - a mathematical process **model**





Kalman Filter

Sensors capture incomplete or noisy information



Kalman Filter

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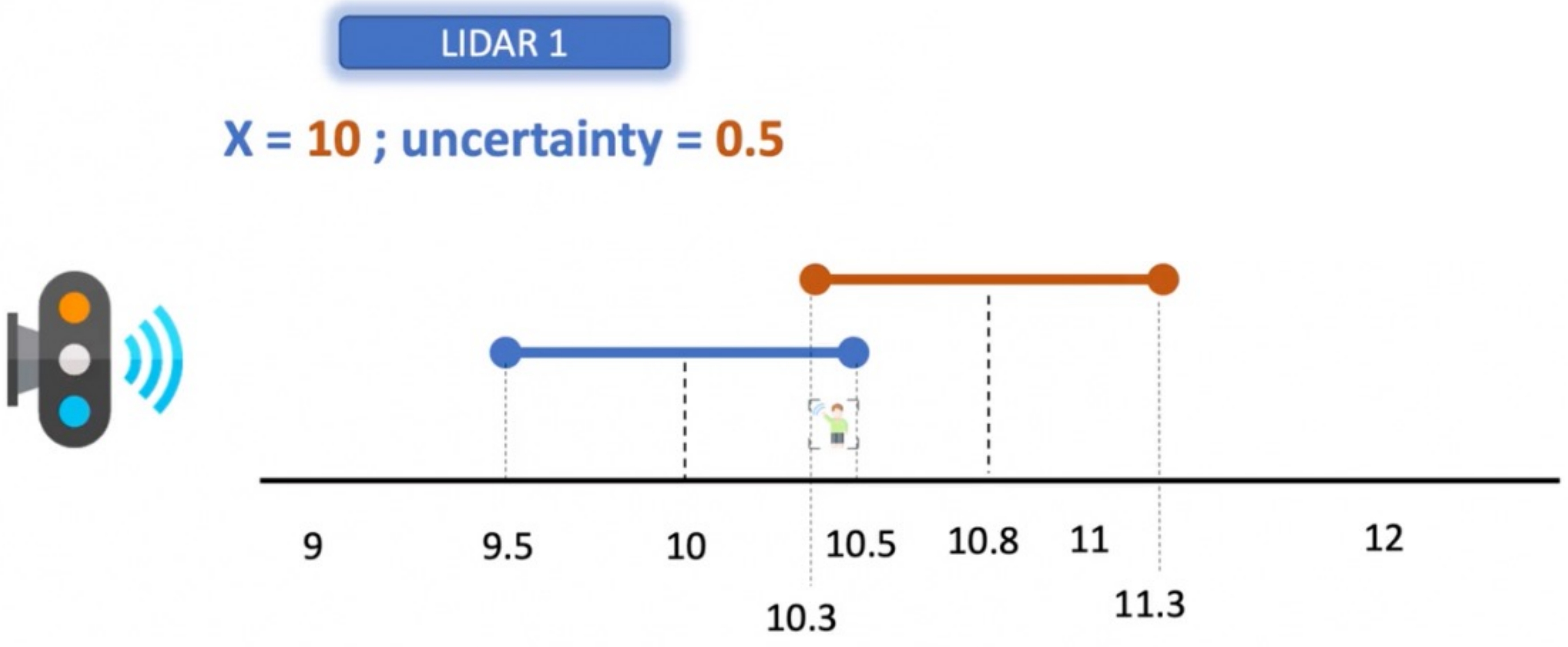

Kalman Filter

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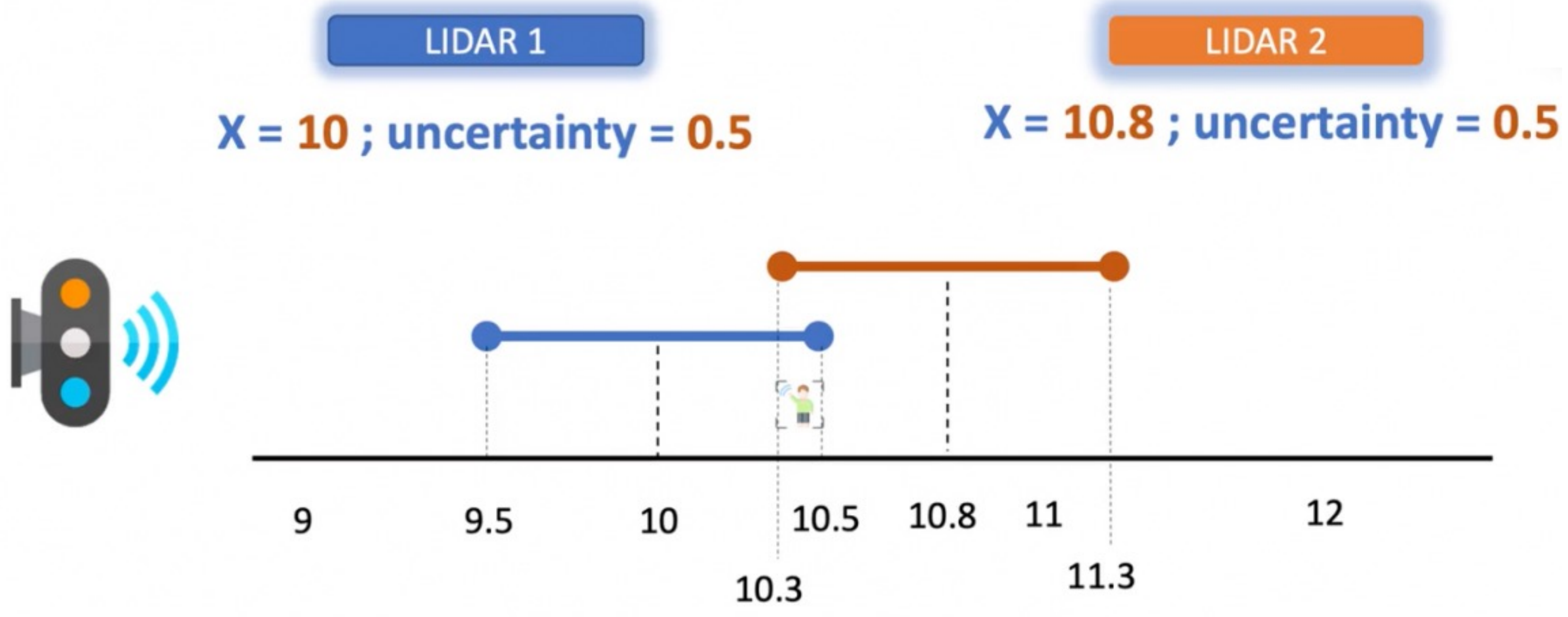

Kalman Filter

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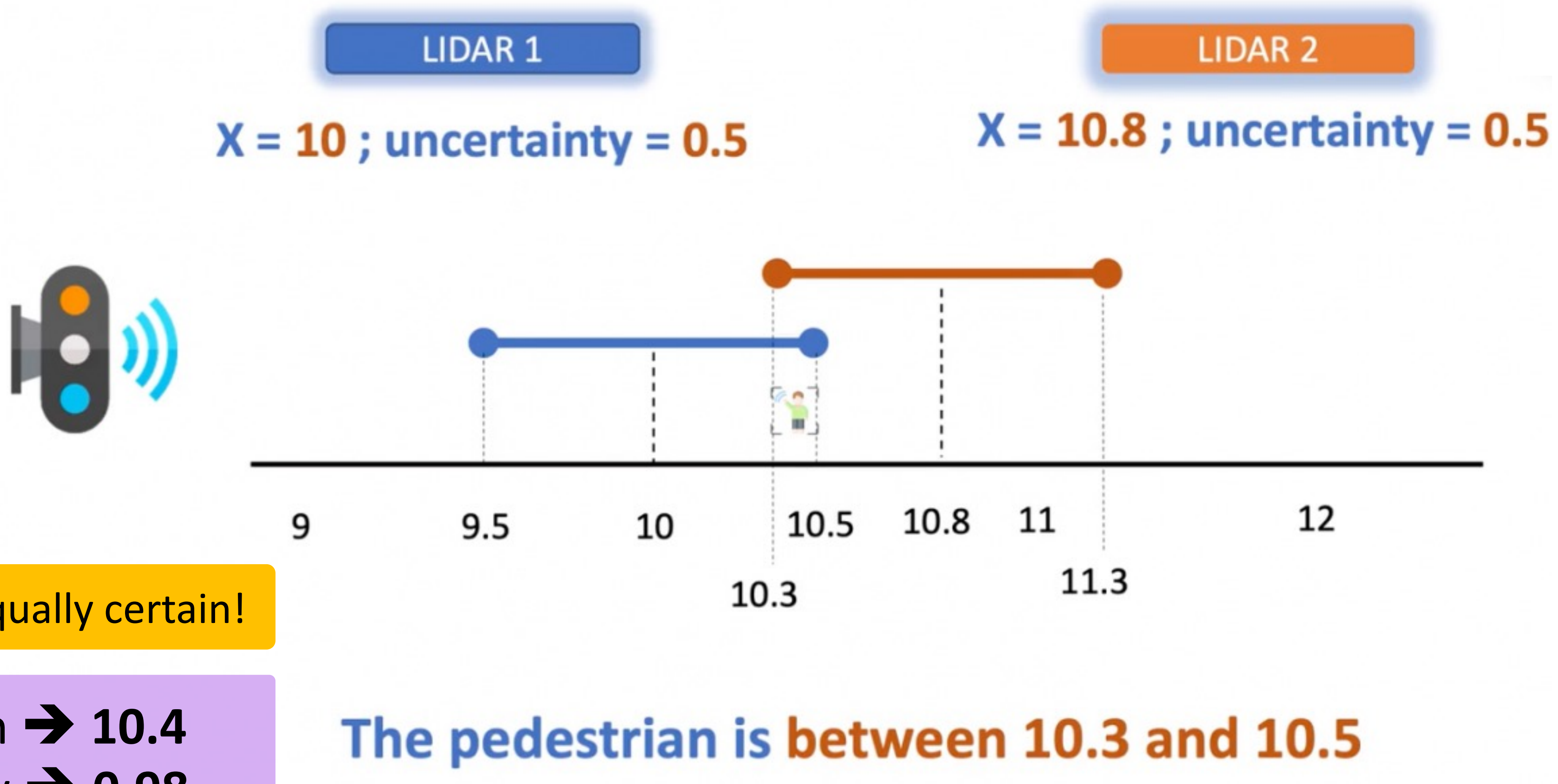
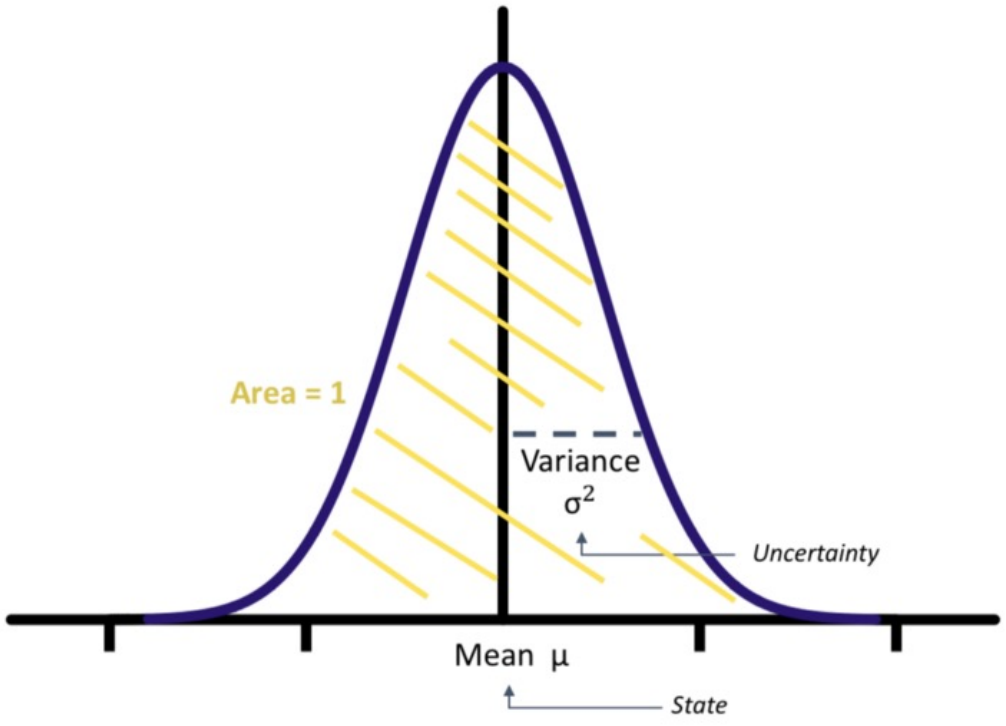


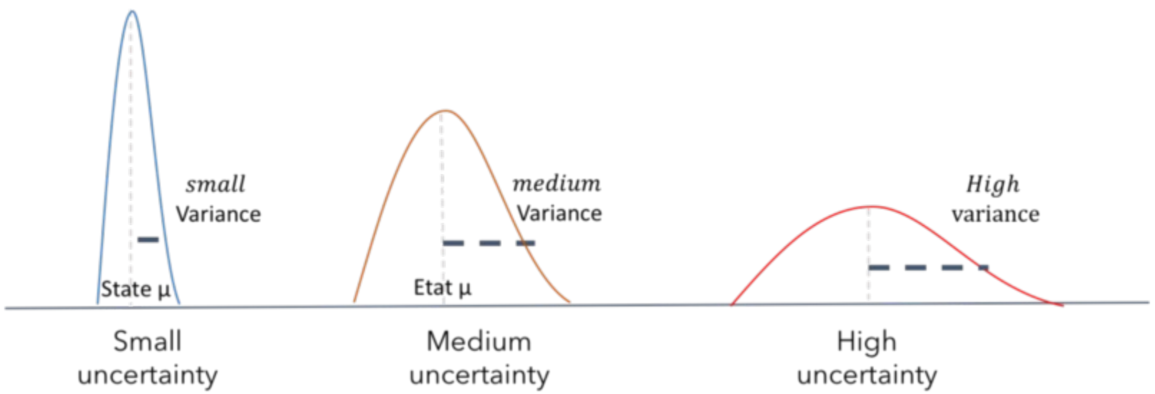
Both sensors equally certain!

Pedestrian → **10.4**
probability → **0.98**

Gaussians

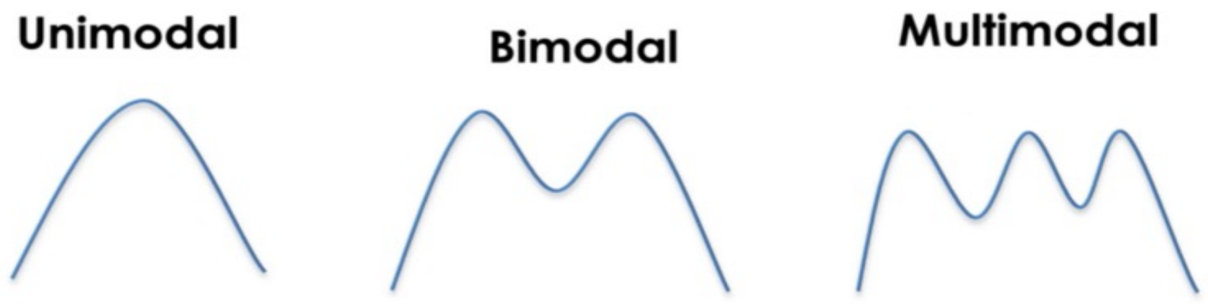
Kalman Filters express state and uncertainty using **Gaussians**





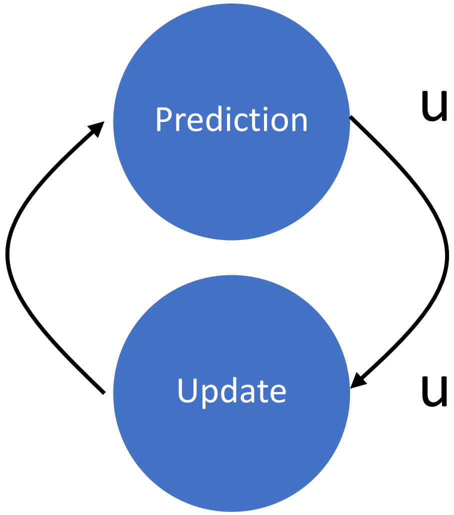
Kalman Filter and Gaussians

- Kalman Filter is unimodal
    - Single peak each time



- An obstacle is **not** both, 10m away with 90% and 8m away with 70% probability
- **9.7m away with 98% or nothing**

Kalman Filter | Under the Hood



use estimated state to **predict future** state+uncertainty

use observations of sensors to **correct** prediction+**improve** accuracy

state of system

$$x = \begin{pmatrix} p \\ v \end{pmatrix}$$

position

velocity

Only needs **current observations** and **previous predictions**

Kalman Filter | Steps

- Imagine sensor fusion between LiDAR and RADAR
- Only their outputs (late fusion)
- Kalman Filter: unimodals and Gaussians to represent state/uncertainty
- Prediction/update cycle

Kalman Filters | Predictions

- estimate state x' and uncertainty P' at time t+1

transition matrix from t to t+1

estimated state of system at t+1

old state of system at time t

$$x' = Fx + u$$

noise

estimated uncertainty of system at t+1

old uncertainty of system at time t

$$P' = FPF^T + Q$$

covariance matrix including noise

Kalman Filter | Predictions [contd.]

- New position = former position (x) times matrix (F)
- F ➔ motion model
  - matrix describing how to move from t to t+1

position (t+1) = position (t) + velocity (t)*time
velocity (t+1) = velocity (t)

constant velocity

- Can consider other motion models for F
  - Constant Turn Rate, Constant Velocity, Constant Acceleration

Kalman Filter | Prediction Matrix Example

$$x' = F \cdot x$$

$$x' = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \text{position} \\ \text{velocity} \end{bmatrix}$$

$$x' = \left| \begin{array}{l} \text{position} + \text{velocity} \cdot \Delta t \\ \dot{x} \end{array} \right.$$

Kalman Filter | Update Measurements

- Adjust position, correct how to update next step

difference between actual measurements and prediction

actual measurement

observation matrix

$$y = z - Hx'$$

prediction

system error

Kalman Gain [between 0 and 1]

$$S = HP'H^T + R$$

$$K = P'H^TS^{-1}$$

sensor noise

Kalman Filter | Final Update

- Compute new x and P

$$x = x' + Ky$$

$$P = (I - KH)P'$$

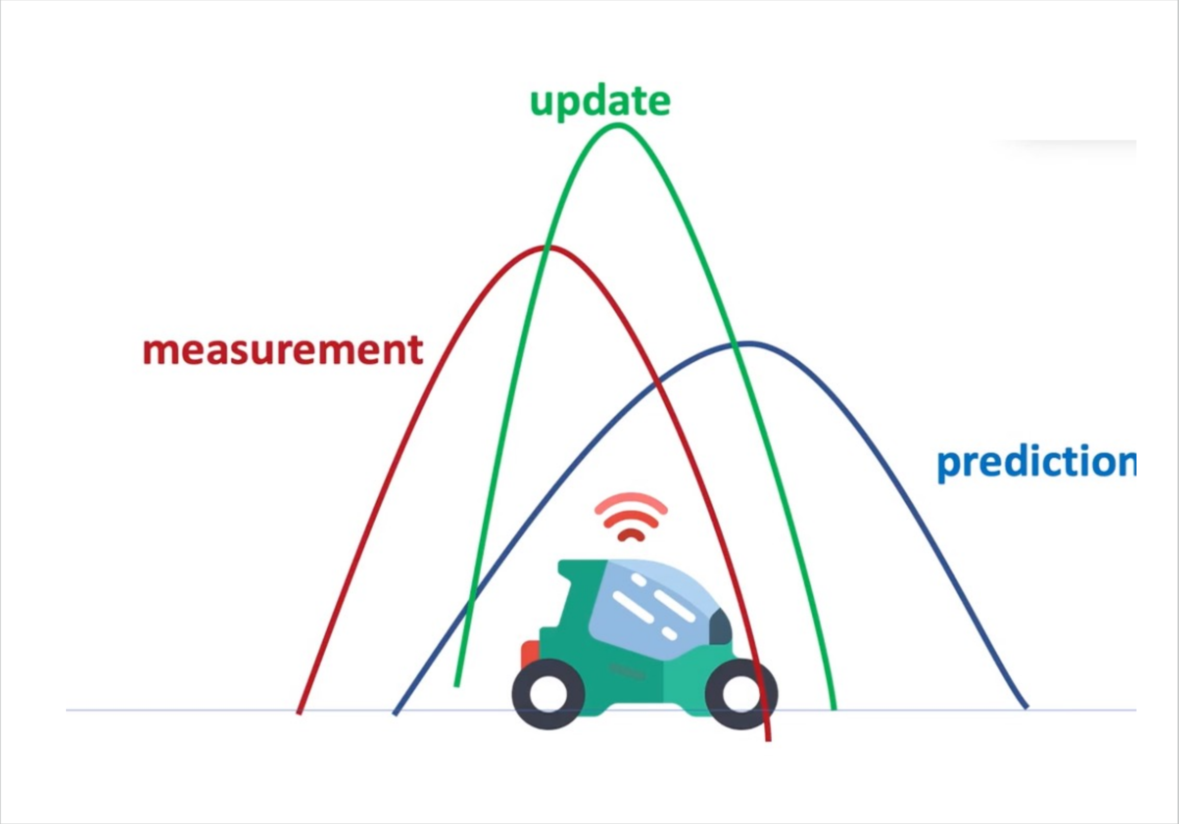
Kalman Filter | Bayesian Filtering

- We want to **estimate posterior**
  - posterior = prior (prediction) * likelihood (measurement)

likelihood prior

$$P(A|B) = \frac{P(B|A) * P(A)}{P(B)}$$

posterior normalizer



RADAR/LiDAR Sensor Fusion Flow

Each time a sensor value comes in ➔ new prediction+update cycle

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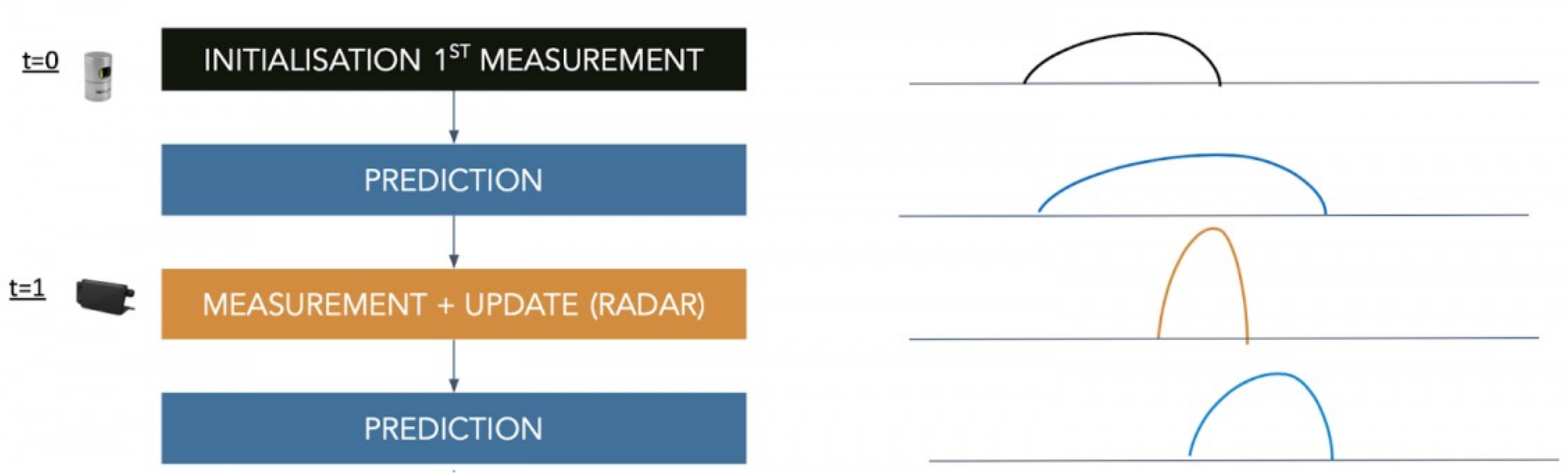
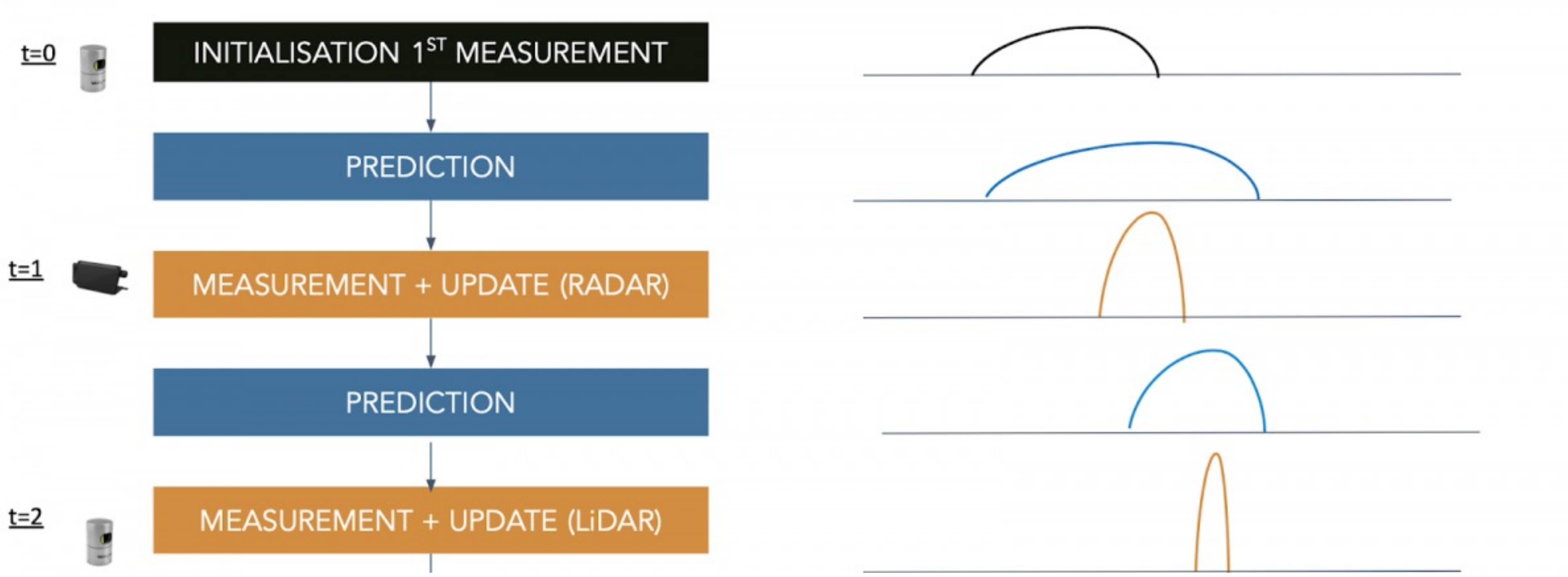


RADAR/LiDAR Sensor Fusion Flow

Each time a sensor value comes in → new prediction+update cycle



RADAR/LiDAR Sensor Fusion Flow

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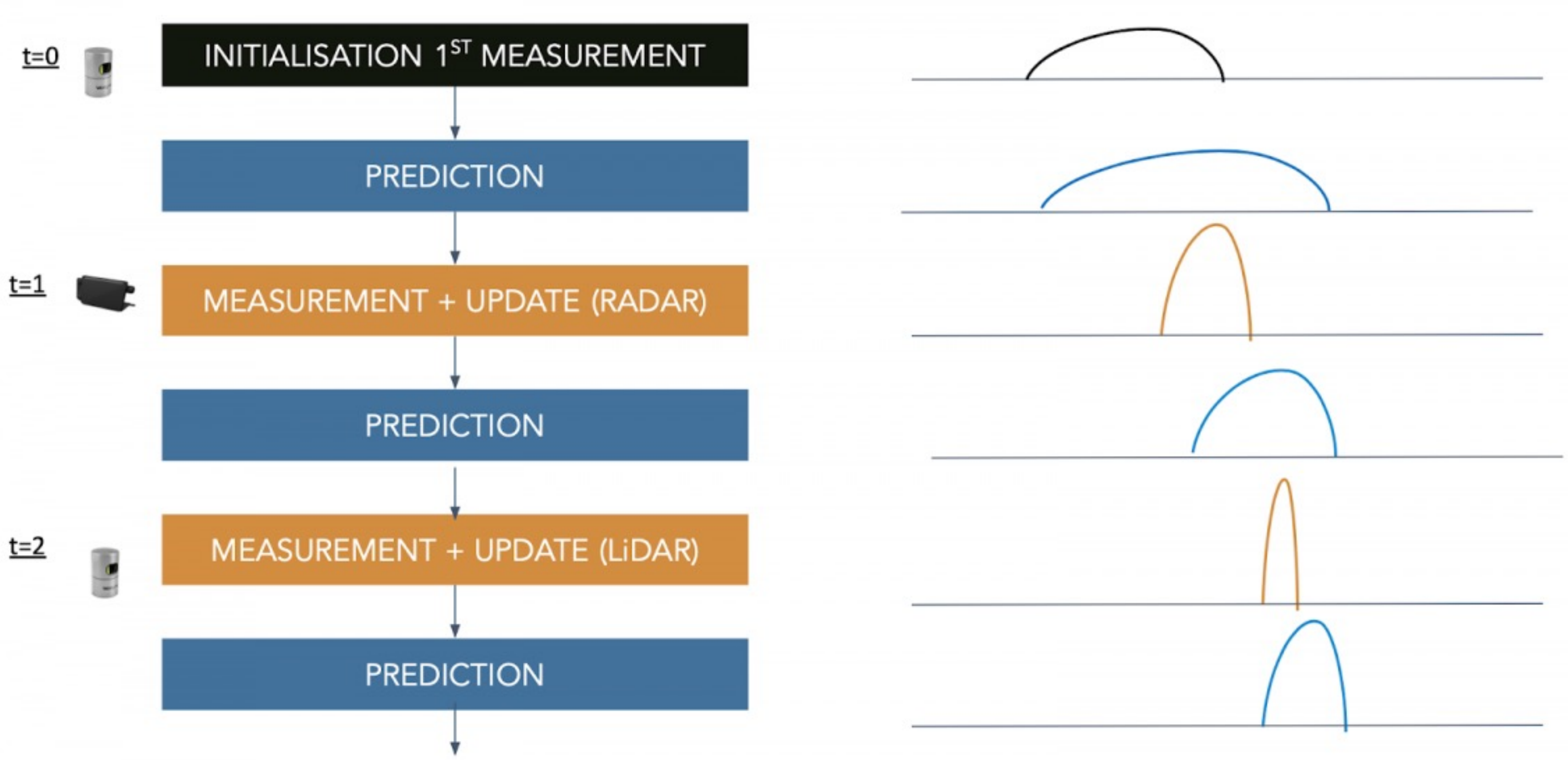
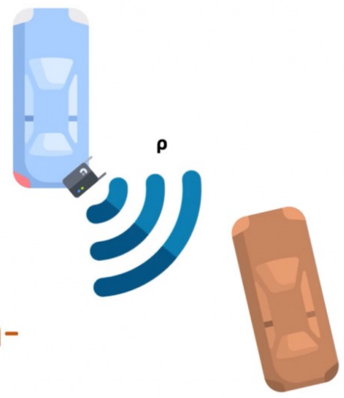


Kalman Filter | Hang on a minute…

- LiDAR has cartesian (linear) values of type y=ax+b
- RADAR → **not linear**!



Measurements are non-linear

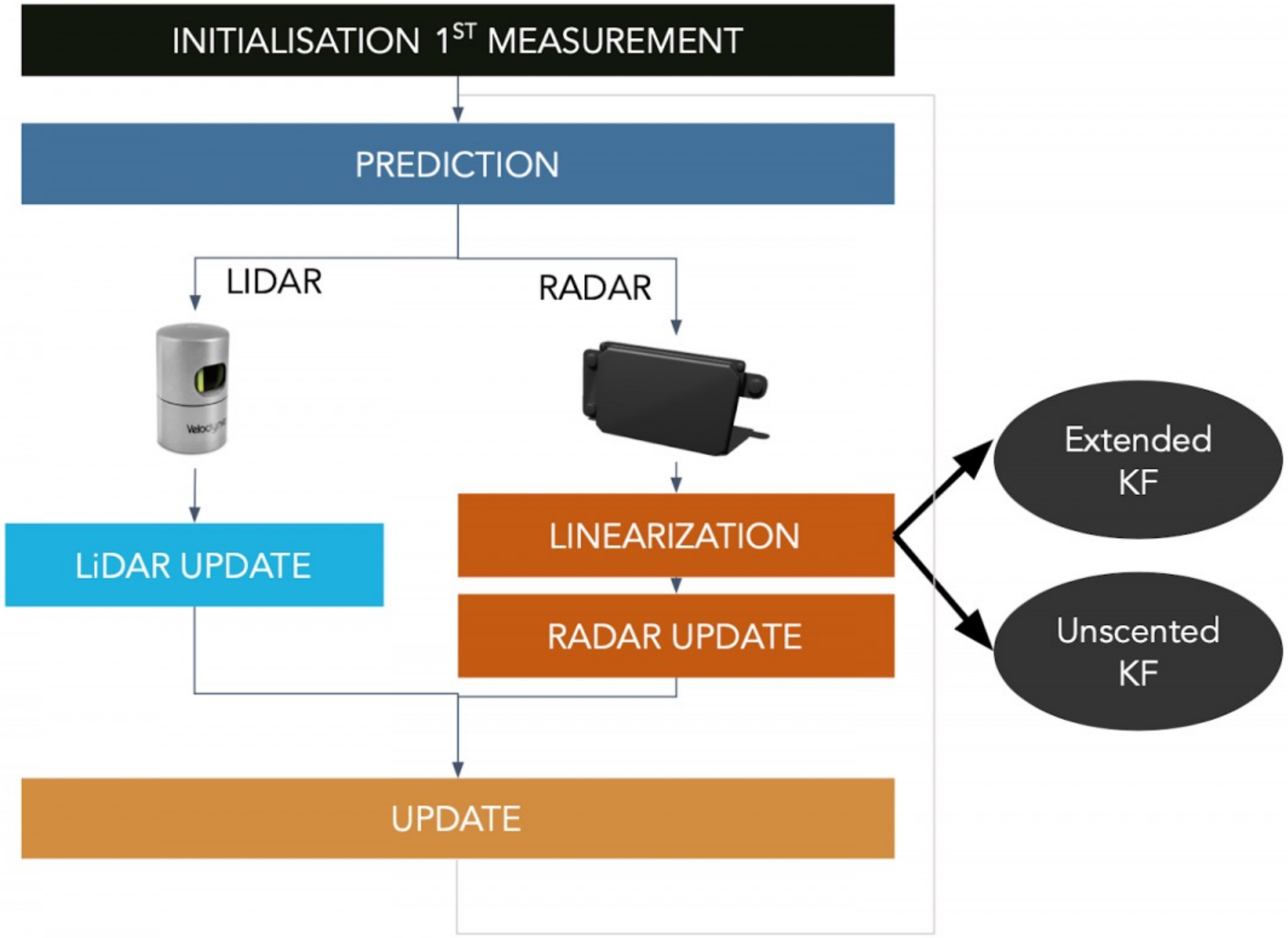
- How to reconcile the two?

Non-Linear Kalman Filters

- The world is **non-linear**
  - Not everything moves in straight lines
  - All sensors work differently

- Two types of Kalman Filters
  - Extended Kalman Filters
  - Unscented Kalman Filters

Final Sensor Fusion Flow



References

- Kalman Filter (with code):

https://towardsdatascience.com/wtf-is-sensor-fusion-part-2-the-good-old-kalman-filter-3642f321440

- A decent primer on the Kalman Filter

https://www.thinkautonomous.ai/blog/?p=sensor-fusion

- Extended Kalman Filter with a nice video

https://kusemanohar.wordpress.com/2020/04/08/sensor-fusion-extended-kalman-filter-ekf/

- Some papers that use EKF
  - https://ieeexplore.ieee.org/document/1338645
  - https://www.ncbi.nlm.nih.gov/pmc/articles/PMC7435659/